Asset Allocation for Robust Portfolios

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Summary

A new risk return optimisation model is described that overcomes much of the instability inherent in Mean Variance optimisers, with minimal sacrifice of efficiency. The Robust Frontier model identifies portfolios that are close to the efficient frontier, less likely to produce extreme results and, most importantly, are much more stable to changes in input assumptions. Practitioners using this model can approach the task of asset allocation with confidence that small errors in forecasts will not be the primary driver of asset allocation outputs and that, if the model is used over time, it will not produce huge swings in allocations with consequent high transaction costs.

Difficulties in implementing Mean Variance style optimisers

Various authors have documented the difficulties that many practitioners have had in implementing Mean Variance (MV) style optimisers. At the heart of these difficulties is the instability inherent in the way the MV models trade off risk and return. Michaud and others have referred to the extreme sensitivity of Mean Variance style optimisers to imput assumptions. Small changes in input variables can have quite profound impacts on the structure of Efficient Portfolios, so much so that Michaud refers to MV optimisers as error maximisers.

This instability is then compounded by the difficulty in developing precise imput data.

- Estimates for future expected returns carry a large measure of uncertainty at the best of times.
- There has been considerable debate over what is the best measure of risk, let alone how one should then estimate that measure. Balzer and others reviewed this issue at some length. Which measure should we use, and how best to gain a forward looking estimate of that measure?
- Correlation coefficients vary over time and in particular tend to increase during times of negative returns (Jacquier & Marcus). What correlation coefficients should we try to estimate, and how do we go about doing that? Again, considerable effort has gone into that area of study (Campbell, Koedijk & Kofman).

The combination of high levels of uncertainty around inputs, and extreme sensitivity of outputs to small changes in those input assumptions, means that often the main determinant of output lies in the unavoidable error terms of the assumptions. As a result practitioners primarily use the tool as a sanity check against their own gut feeling as to what an asset allocation should look like, rather than a primary driver of asset allocation decisions.

Criteria to assess optimisation models

The objective of this study has been to find an optimiser that is superior to the MV model, and in particular one that produces more stable and intuitive results with minimal loss in expected returns.

In order to assess whether the Robust Frontier model is an improvement on the MV style model it is compared across 3 criteria

- Stability; how much do changes in inputs change the outputs of the model?
- Efficiency; how close are expected returns to that of the Efficient Frontier? Efficiency is measured by expressing the returns of a Robust Portfolio as a percentage of the returns available from an efficient portfolio of the same degree of risk.
- Robustness; how likely is it that model portfolios will produce returns that substantially underperform the efficient frontier when viewed from an ex-post perspective?

The Robust Frontier Model

Mean Variance optimisers develop an efficient frontier of portfolios which have the highest expected average return for each level of risk. Just below that efficient frontier there exists a cloud of portfolios that may have very different asset weightings but have very similar risk and return characteristics. This, of course, is the driver of the instability inherent in MV models; small changes in input assumptions can cause a portfolio that was close to the frontier to move to the frontier, displacing the old efficient portfolio. Both portfolios still have very similar risk and return characteristics, one has just moved from being marginally worse to marginally better than the other.

The Robust Frontier Model aims to identify the most robust of those portfolios in the cloud close to the ex ante efficient frontier. That is, those that will perform as close to the ex-post frontier as possible over as wide a range of scenarios as possible. The ex post efficient frontier is the efficient frontier that will be viewed after the fact.

Methodology

1. Estimate Expected Return Distributions

Expected return, risk and correlation matrices for the assets under considerations are required to generate a range of Monte Carlo simulations. Whichever distributions that are believed to be most appropriate to the assets involved may be used. For the purpose of illustrating the use of the model in this paper, normal distributions have been used.

2. Construct Ex-post Efficient Frontiers

To construct ex-post efficient frontiers, Monte Carlo simulations are used to generate a range of possible return scenarios for each of the assets being considered. For each scenario an MV efficient frontier is constructed using the Monte Carlo return for each asset and the ex ante risk measures for each asset.

In construction of the ex post efficient frontier it may seem contradictory to use ex *post* returns and ex *ante* risk measures. There are a number of reasons for doing this

- The author's experience of many years of dealing with private investors and their intermediaries is that, after the fact, investors tend to be primarily concerned with actual returns achieved and the perceived risk incurred at the outset.

- After the fact, actual returns achieved are known, but risk incurred at the outset still remains an unknown quantity. This draws a distinction between volatility of returns and risk. While ex post volatility is measurable, it is not obvious that it actually captures the true risk of the investment at the start of the period. Similarly, actual returns achieved often bear little semblance to the true expected returns at the outset (which of course also remain unknown quantities.)

- Having a stable measure of risk allows portfolios to retain their risk characteristics for comparison across the range of Monte Carlo outcomes.

3. Find Robust Portfolios

In order to identify the robust portfolios, for each scenario the difference in return between each candidate portfolio and the ex-post efficient frontier is then calculated. This is raised to the power of λ.

The higher the value of λ the more portfolios are penalised for producing returns that are distant from the ex post efficient frontier. This is consistent with the commonly observed trait that investors are not particularly concerned about modest underperformance but become increasingly concerned as the size of underperformance increases.

The portfolio that, for a given level of risk, has the lowest sum of differences, raised to the power of λ, across the entire range of scenarios becomes the Robust Portfolio for that risk level. The full set of such portfolios across the range of risk becomes the Robust Frontier.

Consider a two portfolio, two scenario example, where Portfolio 1 is 2% away from an ex-post frontier in both scenarios and Portfolio 2 is 3% away in one scenario and 0%, or on the frontier, in the other scenario. At λ=1, Portfolio 2 has the lower sum of differences ie 4 v 3. However at λ =2, the Portfolio 1 is the preferred portfolio with a sum of differences squared of 8 compared with 9 for Portfolio 2. Thus as λ increases portfolios which are more consistently close to the efficient frontier are preferred.

Mathematically:

If X_i = allocation to asset i
And X_i^e = ex post efficient portfolio allocation to i
And r_{ij} = the j^{th} simulation of the return of i
And λ = a coefficient of sensitivity to underperformance

Then X_i^R is the Robust Portfolio that minimizes

$$\sum_{i,j} (X_i^e \,.\, r_{ij} - X_i \,.\, r_{ij})^{\lambda}$$

for a given level of risk.

Special Case λ=1, The Mean Variance Model

When λ=1 the model reverts back to an approximation of the MV model.

$$X_i^R = \min \sum_{i,j} (X_i^e \,.\, r_{ij} - X_i \,.\, r_{ij})^{\lambda} \quad \text{for a given level of risk}$$

$$= \min \sum_{i,j} (X_i^e \,.\, r_{ij} - X_i \,.\, r_{ij}) \quad \text{for } \lambda=1$$

$$= \max \sum_{i,j} (X_i \,.\, r_{ij})$$

because $\sum_{i,j} (X_i^e \,.\, r_{ij})$ is constant for all candidate portfolios.

Thus X_i^R = MV efficient portfolio where λ=1.

Because MV efficient portfolio = max $\sum (X_i \,.\, r_{ij})$, as this is the highest average return for a given level of risk

Assumptions

In order to illustrate the **Robust Frontier** approach a simple framework is used featuring

- Five asset classes
- Normal distributions
- One period model
- 500 Monte Carlo simulations
- $\lambda = 4$

Table 1: Risk and Return Assumptions
(Assumption set A)

	Asset				
	A	B	C	D	E
Expected Return	10.1%	9.1%	6.0%	4.9%	4.0%
Standard Deviation	21.8%	25.0%	8.3%	5.0%	0.6%
Covariance Matrix					
A	1.00	0.46	0.67	0.35	-0.05
B	0.46	1.00	0.26	0.20	0.00
C	0.67	0.26	1.00	0.33	-0.06
D	0.35	0.20	0.33	1.00	-0.02
E	-0.05	0.00	-0.06	-0.02	1.00

In addition to the return assumptions shown in Table 1 a second set of assumptions and results is shown in the appendix to illustrate how the approach works in other circumstances.

Results

1. Robust portfolios

At first glance Table 2 seems to show that Robust and Efficient portfolios do not tend to be dramatically different. Robust Portfolios do however tend to draw more from the broader set of investment alternatives and have less concentration in the most favoured asset sectors. This tendency to have a more even spread simply reflects the fact that a more even spread produces fewer extreme results, and that extreme results are penalised more by the Robust approach than the mean variance approach.

Table 2. Comparison of Robust and Efficient Portfolios
(Assumption Set A)

Asset Allocation For Portfolios Of Different Risk Levels (Std Dev %)								
	4%		8%		12%		16%	
Asset	Robust	Efficient	Robust	Efficient	Robust	Efficient	Robust	Efficient
A	6.4	9.0	14.1	18.7	31.4	38.0	48.0	57.1
B	6.4	5.2	14.9	10.5	24.0	16.3	33.1	21.7
C	16.6	14.6	36.6	28.0	24.7	26.2	14.4	21.2
D	28.2	23.0	26.9	42.7	19.9	19.4	4.5	0.0
E	42.8	48.2	7.5	0.0	0.0	0.0	0.0	0.0

2. Robust portfolios are less sensitive to changes in assumptions

The real advantage of the Robust portfolio approach is clearly illustrated in Tables 3 and 4; it produces portfolios that are far more stable to small swings in return assumptions. These tables show the change in allocation that would result if the return assumptions are modified in a relatively minor way. The expected returns of assets A and C are increased by 1.0% and 0.9% respectively.

In order to assess the relative stability of the models, a measure is introduced which shows the amount of the portfolio that is changed as a result of the change in imputs. The measure of Instability (ψ) is that portion of the portfolio that would be subject to transactions should the change in model be implemented.

Table 3: Effect on Robust Asset Allocations of Changing Expected Return of Asset B from 9.1% to 10.1% and Asset C from 6.0% to 6.9% (Assumption Set A)

Effect of Changing Return Assumptions at Different Risk Levels								
Risk	**4%**		**8%**		**12%**		**16%**	
Returns	**Base case**	**E_b=+1 E_c=+.9**	**Base case**	**E_b=+1 E_c=+.9**	**Base case**	**E_b=+1 E_c=+.9**	**Base case**	**E_b=+1 E_c=+.9**
A	6.4	3.2	14.1	7.7	31.4	24.9	48.0	43.7
B	6.4	7.1	14.9	15.8	24.0	26.2	33.1	36.1
C	16.6	26.6	36.6	53.3	24.7	44.0	14.4	20.2
D	28.2	20.9	26.9	22.8	19.9	4.9	4.5	0.0
E	42.8	42.2	7.5	0.4	0.0	0.0	0.0	0.0
Instability(ψ)	10.7%		18.6%		21.5%		8.8%	

Table 4: Effect on Efficient Asset Allocations of Changing Expected Return of Asset B from 9.1% to 10.1% and Asset C from 6.0% to 6.9% (Assumption Set A)

Effect of Changing Return Assumptions at Different Risk Levels								
Risk	**4%**		**8%**		**12%**		**16%**	
Returns	**Base case**	**E_b=+1 E_c=+.9**	**Base case**	**E_b=+1 E_c=+.9**	**Base case**	**E_b=+1 E_c=+.9**	**Base case**	**E_b=+1 E_c=+.9**
A	9.0	0.0	18.7	1.3	38.0	25.0	57.1	47.4
B	5.2	6.9	10.5	14.8	16.3	24.5	21.7	32.6
C	14.6	37.6	28.0	75.1	26.2	50.5	21.2	20
D	23.0	12.6	42.7	8.8	19.4	0.0	0.0	0.0
E	48.2	42.9	0.0	0.0	0.0	0.0	0.0	0.0
Instability(ψ)	24.7%		51.3%		32.5%		9.7%	

The results of Table 3 and 4 are summarised in Table 5 which clear shows the greater stability of the Robust approach. As a measure, ψ is dependent on the return and risk assumptions made at the outset as well as the choice of which assumptions to vary and by how much. Table 6 shows the results of a second, quite different, set of return assumptions described in the Appendix, and changes of 1% in expected returns of two assets. Again the robust approach produced much more stable outcomes.

Table 5. Instability of Robust and Efficient Portfolios (Assumptions Set A)

	Instability ψ	
Risk Level	**Robust Portfolio**	**Efficient Portfolio**
4%	10.7%	24.7%
8%	17.6%	51.3%
12%	21.5%	32.5%
16%	8.8%	9.7%

Table 6. Instability of Robust and Efficient Portfolios (Assumption Set B: see Appendix)

Risk Level	Instability ψ Robust Portfolio	Instability ψ Efficient Portfolio
4%	12.3%	3.8%
8%	3.3%	7.6%
12%	6.5%	21.8%
16%	11.2%	34.1%

The clear implications of these tables are;

- The MV model is too volatile. In Table 6, an 8% risk portfolio would require 51.3% of the portfolio to be changed if two of the forecast returns were varied by around 1%. This is clearly unworkable in practice.
- In contrast, the Robust model is far more stable. This characteristic is typical across a range of different assumptions and not confined to the two examples shown in this paper.

Practitioners using the Robust Allocation approach can approach the task of asset allocation in the knowledge that the effect of the inherent inaccuracy involved in risk and return forecasting will not dominate the outputs.

3. Robust portfolios tend to entail modest sacrifices in expected returns.

Table 7: Reduction in Expected Returns from Robust Portfolios

Risk-Level %	E(r) Robust Portfolio %	E(r) Efficient Portfolio %	Difference %	Efficiency %
4	5.32	5.33	.01	99.8
8	6.62	6.65	.03	99.5
12	7.85	7.89	.04	99.5
16	8.98	9.05	.07	99.2

The expected returns from the Robust and Efficient portfolios shown in Table 7 do not differ greatly in expected returns. This reflects the large numbers of portfolios with very similar risk/return characteristics that sit very close to the actual efficient frontier. Accordingly it is possible to create far more diversified portfolios without giving up much in the way of expected returns.

Table 8 shows the results obtained using the Assumption Set B. These assumptions also produce portfolios which are highly efficient, albeit slightly less so than the portfolios resulting from Assumption Set A.

Table 8: Reduction in Expected Returns from Robust Portfolios (Assumption Set B)

Risk-Level %	E(r) Robust Portfolio %	E(r) Mean Risk Portfolio %	Difference %	Efficiency %
4	5.50	5.53	0.02%	99.6%
8	7.02	7.07	0.05%	99.3%
12	8.43	8.57	0.14%	98.4%
16	9.31	9.51	0.20%	97.9%

4. Robust portfolios are less likely to produce extreme results

Table 9 shows how the Efficient and Robust portfolios produce results away from the ex post efficient frontier.

Table 9. Robustness of results: Proportion of results departing from the Ex Post Efficient Frontier by more than given %

Departure from frontier	Risk 4%		8%		12%		16%	
	Robust	Efficient	Robust	Efficient	Robust	Efficient	Robust	Efficient
>1%	35.6%	33.3%	67.1%	62.6%	67.9%	67.7%	64.8%	62.8%
>2%	2.9%	3.7%	26.6%	22.1%	33.7%	30.3%	43.6%	34.8%
>3%	0.2%	0.2%	6.7%	7.4%	13.7%	11.7%	22.5%	17.6%
>4%	0.0%	0.0%	0.4%	1.8%	6.3%	6.3%	10.4%	10.4%
>5%	0.0%	0.0%	0.2%	0.4%	1.0%	3.9%	5.7%	6.1%

The difference in spread of outcomes is not as marked as perhaps one might expect. However, given that the efficient portfolio is on *average* the closest to the efficient frontier and the Robust portfolio has fewer outlying results, it should be expected that the efficient portfolio will have a higher proportion of results close to the frontier than the Robust portfolio. The point at which the Robust Portfolios begin to produce fewer outliers tends to be at the 1 to 5 percentile level, the really extreme results.

These levels are related to the level of risk of the portfolio, a 5 percentile outlier is obviously closer to the efficient frontier for a 4% risk portfolio than a 16% risk portfolio.

The Effect of Changing the Value of λ

Increasing the value of λ increases the penalty to candidate portfolios to variation from the ex-post frontier; hence the higher the value of λ, the greater the diversity and stability of the portfolio. However this does come at a modest cost to efficiency. Table 10 shows the effect of increasing λ from 1 through to 8.

Table 10. Effect of Changing λ on Portfolio Characteristics (Assumption Set A)

Risk Level	λ	Efficiency	Instability	Frequency of Departure From Frontier >1%	>2%	>3%	>4%	>5%
4%	Efficient Portfolio	100.0%	24.4%	24.1%	3.3%	0.2%	0.0%	0.0%
	1	99.9%	24.8%	23.9%	3.3%	0.2%	0.0%	0.0%
	2	99.9%	21.2%	24.9%	2.5%	0.2%	0.0%	0.0%
	4	99.8%	14.1%	26.8%	1.8%	0.0%	0.0%	0.0%
	8	99.7%	11.2%	27.2%	1.6%	0.0%	0.0%	0.0%
8%	Efficient Portfolio	100.0%	54.9%	44.6%	18.0%	5.1%	1.6%	0.4%
	1	100.0%	53.4%	44.2%	18.2%	5.1%	1.6%	0.4%
	2	99.9%	45.2%	46.4%	18.8%	4.7%	0.8%	0.0%
	4	99.5%	29.6%	48.5%	20.2%	4.7%	0.6%	0.0%
	8	99.1%	21.1%	50.5%	19.6%	4.5%	0.6%	0.0%
12%	Efficient Portfolio	100.0%	57.4%	45.0%	25.4%	14.5%	7.8%	4.5%
	1	100.0%	62.3%	45.0%	25.4%	14.5%	7.8%	4.5%
	2	99.8%	40.5%	55.2%	27.8%	11.7%	4.1%	1.0%
	4	99.5%	33.0%	58.9%	28.4%	12.1%	2.9%	1.0%
	8	99.4%	26.3%	62.2%	29.2%	12.3%	3.7%	1.2%
16%	Efficient Portfolio	100.0%	57.1%	43.4%	28.8%	22.5%	14.9%	11.5%
	1	99.9%	67.0%	43.4%	28.8%	22.5%	14.9%	11.5%
	2	99.6%	29.8%	67.3%	40.5%	21.5%	11.2%	4.7%
	4	99.2%	20.1%	68.9%	42.1%	21.9%	11.0%	5.3%
	8	99.2%	11.2%	70.1%	43.1%	24.3%	11.0%	6.1%

As can be seen there are normally substantial increases in stability to be achieved by increasing λ from 1 through to 4 with only modest sacrifice of Efficiency. Beyond that there are further, more modest gains in stability to be achieved, however at λ=8 the robustness of the portfolios begins to deteriorate. This is again witnessed in Table 15 (Appendix.) Accordingly, λ=4 has been chosen as the model input.

The other point to note is that at λ=1 the portfolios are not identical to the Efficient Portfolio because the Robust Portfolio approach is an approximation due to the random nature of the Monte Carlo process.

Conclusion

The Robust Portfolio provides a logical methodology for practitioners to trade off risk and return when making asset allocation decisions. It does so in a way that produces portfolios that have expected risk/return outcomes close to MV efficient portfolios, but with more diversification and less sensitivity to small changes in input assumptions.

It is a methodology that can be used in to build portfolios that will perform well over a wide range of scenarios.

References

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Appendix

Results from the second set of assumptions referred to in the body of the paper.

Table 11: Risk and Return Assumptions
(Assumption set B)

	Asset				
	A	B	C	D	E
Expected Return	9.50%	7.00%	8.00%	13.00%	4.00%
Standard Deviation	19.5%	23.4%	13.6%	32.7%	0.6%
Covariance Matrix					
A	1.000	0.193	0.307	0.871	0.056
B	0.193	1.000	0.050	0.156	-0.007
C	0.307	0.050	1.000	0.726	-0.023
D	0.871	0.156	0.726	1.000	0.029
E	0.056	-0.007	-0.023	0.029	1.000
Changes to E(r)	-1.0			+1.0	

Table 12. Comparison of Robust and Efficient Portfolios
(Assumption Set B)

Asset Allocation For Portfolios Of Different Risk Levels (Std Dev %)								
	4%		8%		12%		16%	
Asset	Robust	Efficient	Robust	Efficient	Robust	Efficient	Robust	Efficient
A	9.9%	11.5%	20.2%	23.3%	26.3%	42.1%	18.9%	34.1%
B	6.9%	3.9%	13.9%	7.8%	20.4%	6.5%	29.3%	6.0%
C	18.7%	19.4%	37.4%	38.8%	48.5%	51.4%	25.5%	38.8%
D	0.0%	0.0%	0.0%	0.0%	4.8%	0.0%	26.4%	21.2%
E	64.4%	65.2%	28.5%	30.0%	0.0%	0.0%	0.0%	0.0%

Table 13: Effect on Robust Asset Allocations of Changing Expected Return of Asset A from 9.5% to 8.5% and Asset D from 13.0% to 14.0% (Assumption Set B)

Effect of Changing Return Assumptions at Different Risk Levels								
Risk	**4%**		**8%**		**12%**		**16%**	
Returns	**Base case**	**E_a=-1 E_d=+1**	**Base case**	**E_a=-1 E_d=+1**	**Base case**	**E_a=-1 E_d=+1**	**Base case**	**E_a=-1 E_d=+1**
A	9.9%	2.3%	20.2%	17.5%	26.3%	19.8%	18.9%	9.9%
B	6.9%	7.4%	13.9%	14.9%	20.4%	22.0%	29.3%	6.9%
C	18.7%	14.0%	37.4%	39.8%	48.5%	52.4%	25.5%	18.7%
D	0.0%	5.1%	0.0%	0.0%	4.8%	5.8%	26.4%	0.0%
E	64.4%	71.2%	28.5%	27.8%	0.0%	0.0%	0.0%	64.4%
Instability(ψ)	12.3%		3.3%		6.5%		11.2%	

Table 14: Effect on Efficient Asset Allocations of Changing Expected Return of Asset A from 9.5% to 8.5% and Asset D from 13.0% to 14.0% (Assumption Set B)

Effect of Changing Return Assumptions at Different Risk Levels								
Risk	**4%**		**8%**		**12%**		**16%**	
Returns	**Base case**	**E_a=-1 E_d=+1**	**Base case**	**E_a=-1 E_d=+1**	**Base case**	**E_a=-1 E_d=+1**	**Base case**	**E_a=-1 E_d=+1**
A	11.5%	8.6%	23.3%	17.6%	42.1%	20.4%	34.1%	0.0%
B	3.9%	4.8%	7.8%	9.5%	6.5%	13.9%	6.0%	18.3%
C	19.4%	22.4%	38.8%	44.7%	51.4%	60.4%	38.8%	51.3%
D	0.0%	0.0%	0.0%	0.0%	0.0%	5.4%	21.2%	30.5%
E	65.2%	64.3%	30.0%	28.2%	0.0%	0.0%	0.0%	0.0%
Instability(ψ)	3.8%		7.6%		21.8%		34.1%	

Table 15. Effect of Changing λ on Portfolio Characteristics (Assumption Set B)

Risk Level	λ	Efficiency	Instability	Frequency of Departure From Frontier				
				>1%	>2%	>3%	>4%	>5%
4%	Efficient Portfolio	100.0%	3.8%	28.0%	2.0%	0.8%	0.0%	0.0%
	1	100.0%	4.7%	27.4%	2.9%	0.8%	0.0%	0.0%
	2	99.9%	3.1%	27.8%	2.0%	0.6%	0.0%	0.0%
	4	99.6%	12.3%	27.8%	2.9%	0.2%	0.0%	0.0%
	8	99.1%	16.7%	29.4%	4.1%	0.0%	0.0%	0.0%
8%	Efficient Portfolio	100.0%	7.6%	52.8%	28.8%	9.0%	2.2%	1.4%
	1	100.0%	7.6%	52.8%	28.6%	9.0%	2.2%	1.4%
	2	99.9%	4.8%	54.4%	27.8%	8.6%	2.5%	1.0%
	4	99.3%	3.3%	55.8%	28.4%	9.2%	3.1%	0.6%
	8	98.6%	3.0%	60.5%	29.7%	10.8%	4.1%	0.8%
12%	Efficient Portfolio	100.0%	21.8%	64.0%	41.9%	27.0%	15.7%	7.6%
	1	99.9%	19.6%	62.6%	42.9%	26.6%	16.0%	8.6%
	2	99.3%	11.5%	69.5%	45.6%	28.2%	15.7%	5.5%
	4	98.4%	6.5%	73.2%	46.6%	29.2%	15.5%	6.3%
	8	97.5%	5.0%	74.4%	49.5%	31.5%	15.5%	8.0%
16%	Efficient Portfolio	100.0%	34.1%	73.4%	50.5%	37.2%	22.7%	14.7%
	1	100.0%	34.1%	73.4%	50.7%	37.2%	22.7%	14.9%
	2	99.2%	17.1%	81.0%	56.9%	39.5%	25.8%	15.1%
	4	97.9%	11.2%	82.6%	62.6%	42.5%	28.8%	16.6%
	8	96.7%	9.1%	82.6%	63.8%	43.4%	30.1%	17.4%